

# **AI-Systems** **Efficient ML** **(294-162)**

Amir Gholami & Joseph E. Gonzalez

# Acknowledgments

Many slides from Jonathan Frankle, Prof. Kurt Keutzer, Pallas Group

# Agenda for Today

- 1:10-2:00: Efficient ML and Ideas in ML Frameworks
- 2:00-2:45: PC Meeting Discussions
- 2:45-3:00: Break
- 3:00-4:00: Guest Lecture by Tianqi Chen

# Objectives For Today

- Get a good understanding of Pruning:
  - Methods for pruning applied to Inference
  - Pruning for efficient Training (Lottery Hypothesis Ticket)
- Learn About Quantization
  - Integer-only Quantization
  - Mixed Precision Quantization
- Important Ideas in ML Frameworks



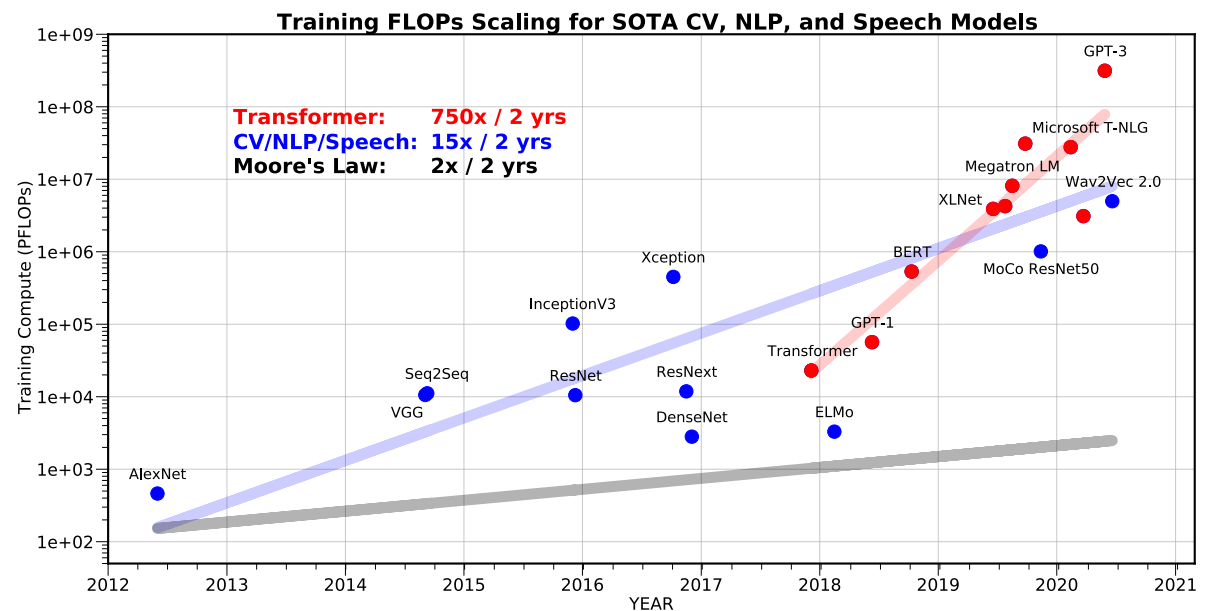
# Efficient ML with Pruning

Most of the slides in this section are courtesy of Jonathan Frankle  
[Harvard/MosaicML]

# Neural Network Training is Costly

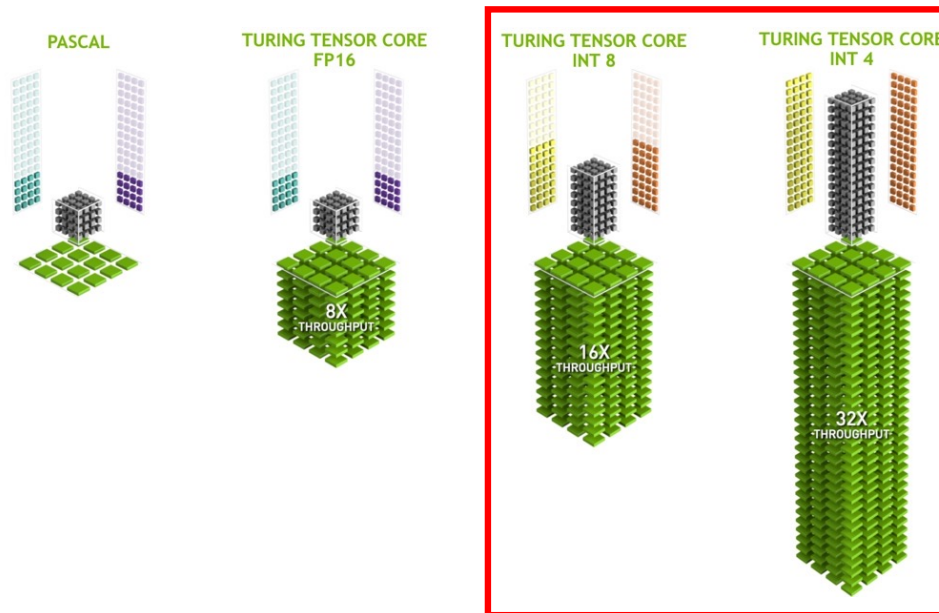
## Research Questions:

- Fundamentally do we need such large models to learn the representations that we are looking for?
- Can we reduce this cost without sacrificing accuracy?



# Reduced Precision Training

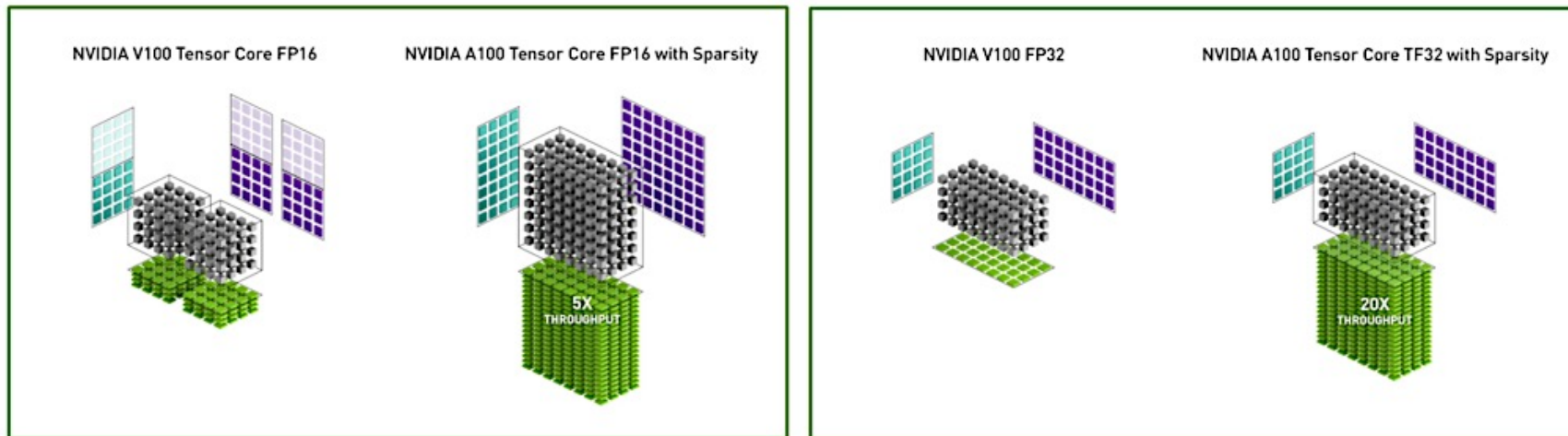
- As we saw in lec 2, we have been very successful in FP16 training which enabled 10x improvements in throughput
  - But it has been very difficult to go to reduce the precision further



INT8/4 training often leads to significant accuracy degradation and is still an open research problem

# How about Utilizing Sparsity?

- New HW generations such as A100 have built-in support for accelerating sparse workloads
  - HW vendors can achieve significantly higher gains if sparse training becomes feasible for ML like FP16



# Sparsity: One of the Challenging Problems in ML

1990

## *Optimal Brain Damage*

---

Yann Le Cun, John S. Denker and Sara A. Solla  
AT&T Bell Laboratories, Holmdel, N. J. 07733

### **ABSTRACT**

We have used information-theoretic ideas to derive a class of practical and nearly optimal schemes for adapting the size of a neural network. By removing unimportant weights from a network, several improvements can be expected: better generalization, fewer training examples required, and improved speed of learning and/or classification. The basic idea is to use second-derivative information to make a tradeoff between network complexity and training set error. Experiments confirm the usefulness of the methods on a real-world application.

# Background: Neural Network Pruning

prune /pru:n/ verb

To reduce the extent of [a neural network] by removing **superfluous** or unwanted parts.

(Oxford English Dictionary)

Goal: reduce the cost of inference  
and/or Training

# Background: Neural Network Pruning

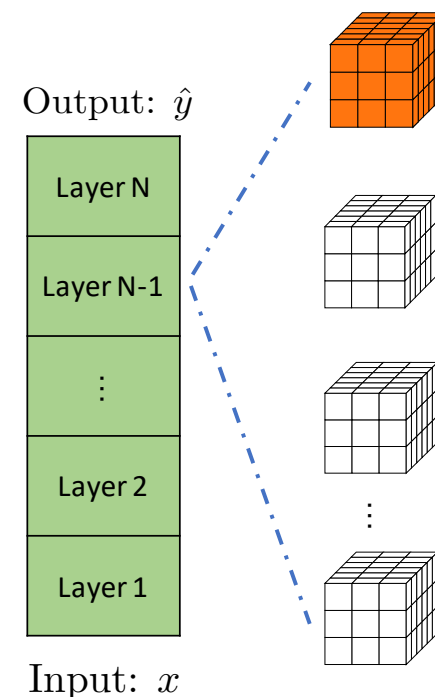
Pruning similar to the **Jenga** game. We only remove blocks that are not sensitive.

Popular metrics for measuring sensitivity:

- Magnitude
- Gradients
- Hessian



[Image from UniversityCoop](#)



Yu S\*, Yao Z\*, Gholami\* A, Dong Z\*, Mahoney MW, Keutzer K. Hessian-Aware Pruning and Optimal Neural Implant. WACV, 2022.  
LeCun Y, Denker J, Solla S. Optimal brain damage. Advances in neural information processing systems. 1989.

# Background: Neural Network Pruning

Pruning similar to the **Jenga** game. We only remove blocks that are not sensitive.

There are many metrics for measuring sensitivity:

- Magnitude
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- Hessian



[Image from UniversityCoop](#)

## Learning both Weights and Connections for Efficient Neural Networks

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Stanford University

John Tran  
NVIDIA

Jeff Pool  
NVIDIA

William J. Dally  
Stanford University

## COMPARING REWINDING AND FINE-TUNING IN NEURAL NETWORK PRUNING

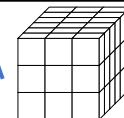
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Michael Carbin  
MIT CSAIL  
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Layer 1

Input:  $x$



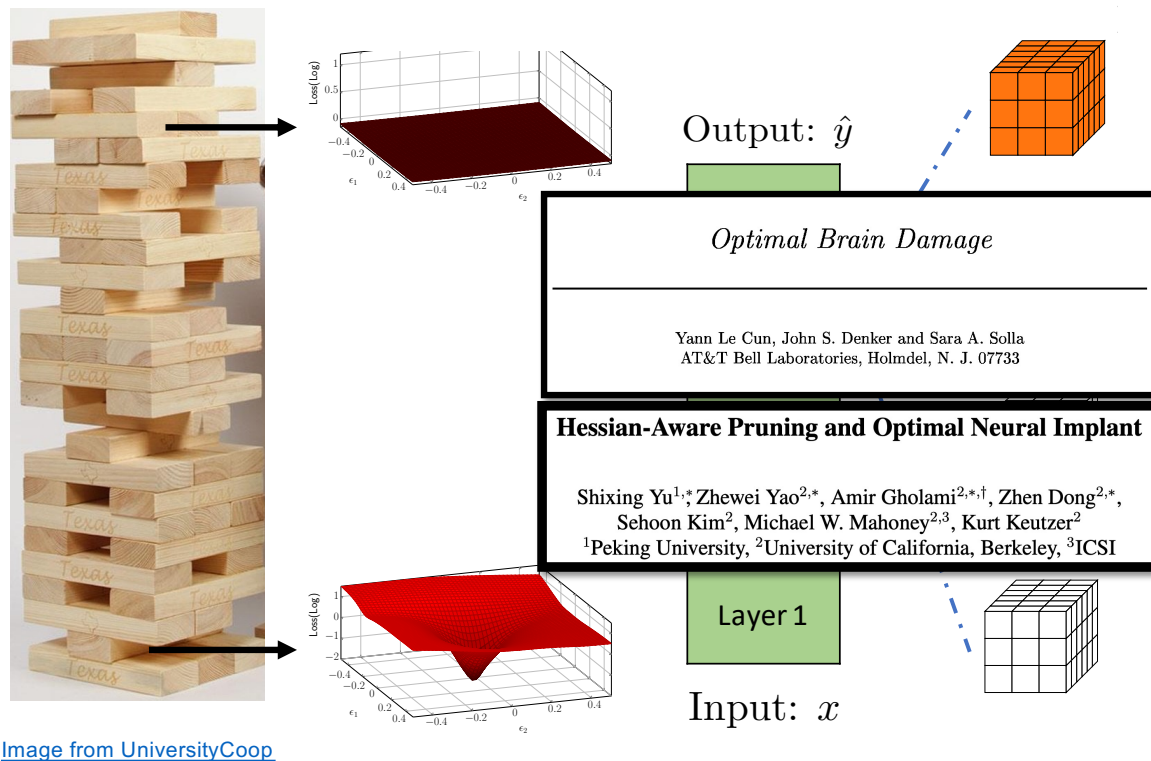


# Background: Neural Network Pruning

Pruning similar to the **Jenga** game. We only remove blocks that are not sensitive.

There are many metrics for measuring sensitivity:

- Magnitude
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# Hessian Aware Pruning

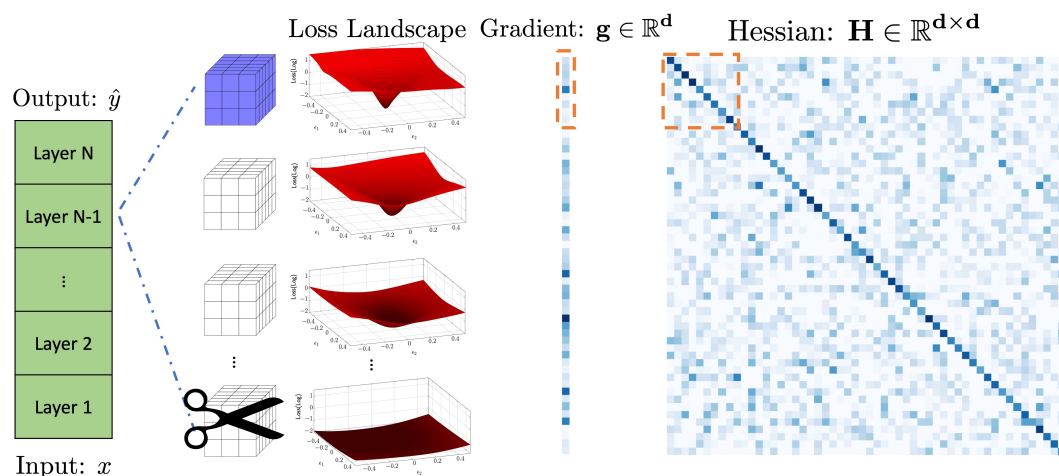
➤ Why Second-order instead of magnitude pruning?

$$L(w) = \frac{1}{N} \sum_{i=1}^N l(x_i, y_i, w)$$

$$\Delta L = L(w + \Delta w) - L(w)$$

$$\Delta L \approx g^T \Delta w + \frac{1}{2} \Delta w^T H \Delta w$$

$$\Delta L = \frac{\text{Trace}(H_{p,p})}{2p} \|w_p\|_2^2$$



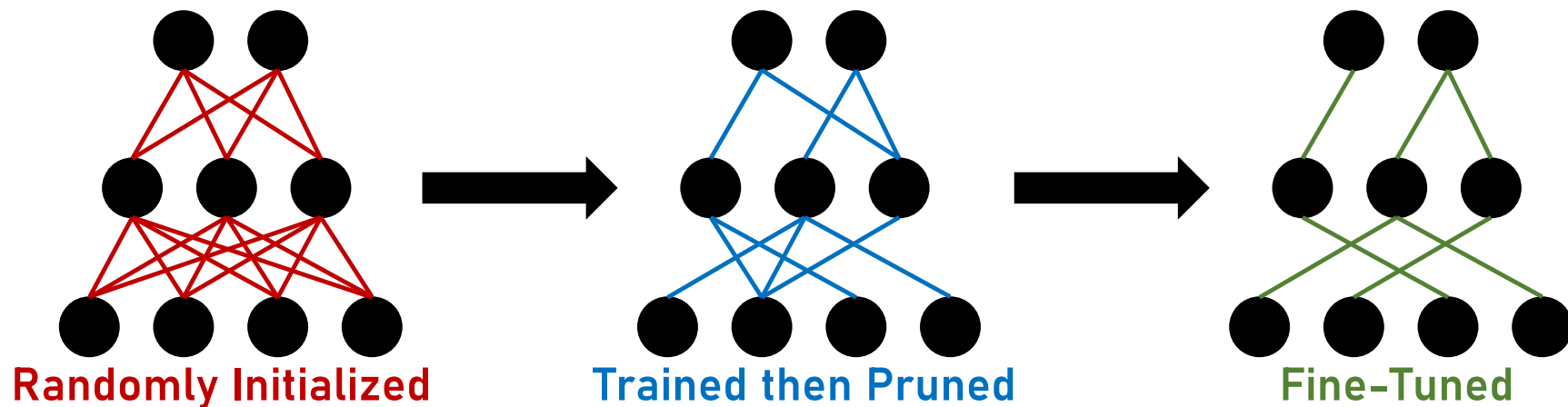
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Yu S\*, Yao Z\*, Gholami\* A, Dong Z\*, Mahoney MW, Keutzer K. Hessian-Aware Pruning and Optimal Neural Implant. WACV, 2022.

Yao Z\*, Gholami A\*, Keutzer K, Mahoney M. PyHessian: Neural networks through the lens of the Hessian. AAAI, 2020.

# Background: Neural Network Pruning

- 1) Train the network
- 2) Remove superfluous structure
- 3) Fine-tune the network
- 4) Optionally: prune and fine-tune iteratively



# Pruning Can be Effective

- If we can indeed find pruned models that achieve high accuracy that means that there exists sub-networks that have enough representational power.
- **Then why not sparse models to begin with?**

Different  
structured  
pruning  
methods

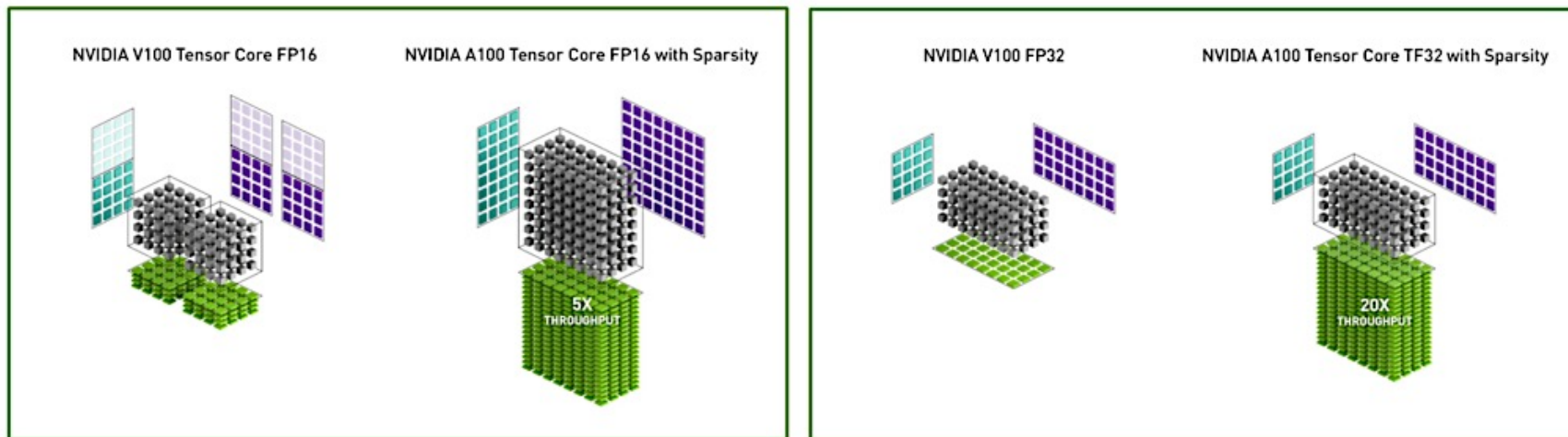
Method	Acc.(%)	Param.(%)	FLOPs(%)
VGG16	93.96	100.0	100.0
L1[36]	93.40	36.0	65.7
SSS[25]	93.02	26.2	58.4
VarP[73]	93.18	26.7	60.9
HRank[37]	93.43	17.1	46.5
GAL-0.05[39]	92.03	22.4	60.4
HRank[37]	92.34	17.9	34.7
GAL-0.1[39]	90.73	17.8	54.8
HAP	<b>93.66</b>	<b>10.1</b>	<b>29.7</b>

# Research Question

Research Question: If we can prune models after training, can we train smaller models?

# Research Question: Training Sparse Models?

If we could train small models that are sparse from the beginning, then we can significantly reduce the cost of training



# Research Question

If we can prune models after training, can we train smaller models?

Capacity for representation vs. optimization

# Research Question

If we can prune models after training, can we train smaller models?

Capacity for representation vs. optimization



# Research Question

If we can prune models after training, can we train smaller models?

Capacity for representation vs. **optimization ??**

# Overparameterized Models

- Current Hypothesis: Empirical results have consistently been showing that large models are easier to train and generalize better
  - It appears as if adding more parameters makes the optimization problem easier
  - If this is true, then is training small models infeasible?



Adding more parameters

(Disclaimer: This hypothesis is highly debated and is not always true!)

# Is Training Small-Scale NNs Possible?

ICLR 2019

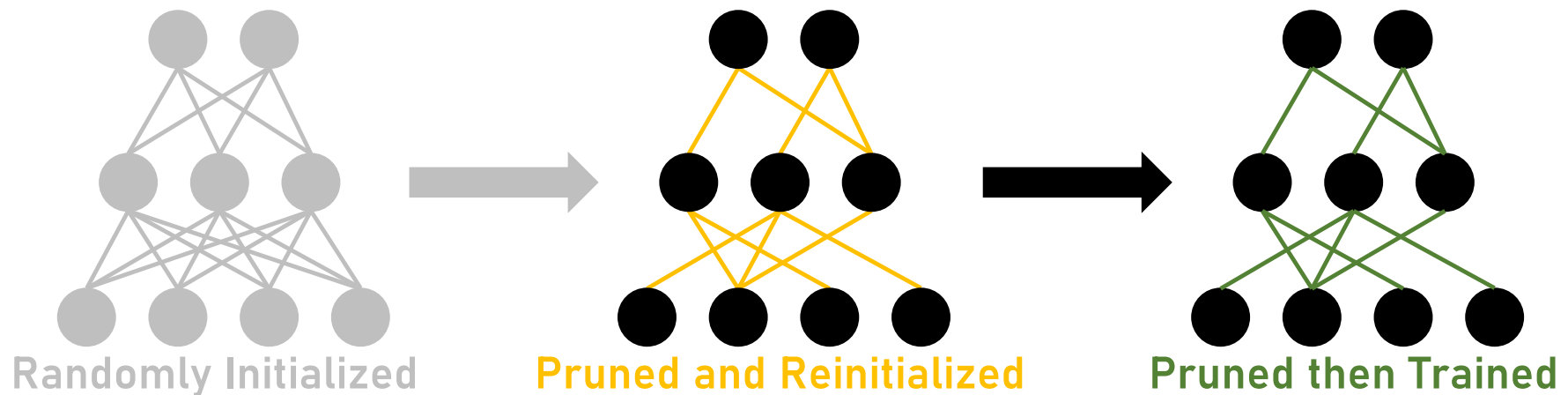
## THE LOTTERY TICKET HYPOTHESIS: FINDING SPARSE, TRAINABLE NEURAL NETWORKS

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# Attempting to Train a Pruned Network

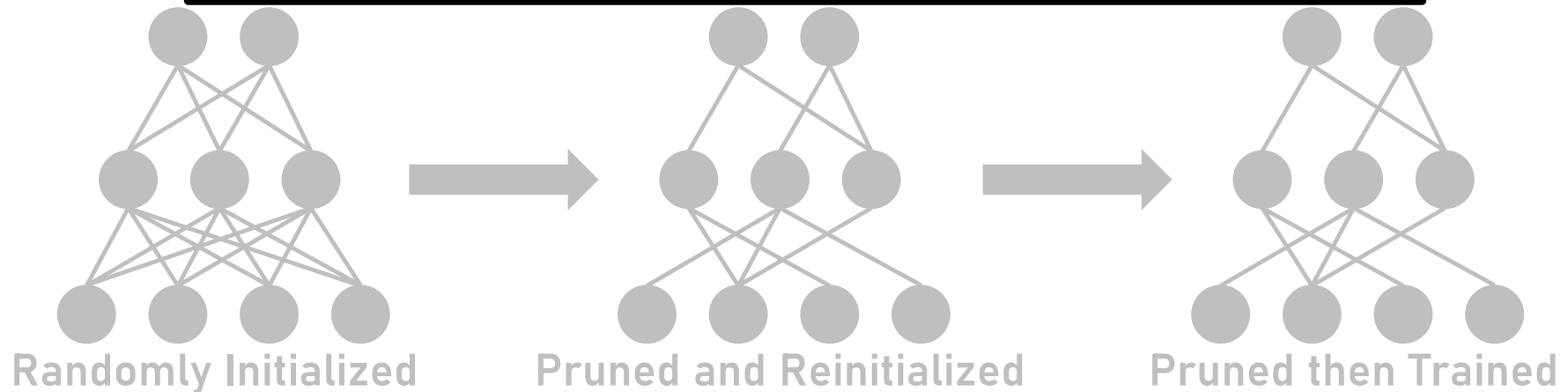
- 1) Train the network
- 2) Remove superfluous structure
- 3) Randomly initialize the pruned network
- 4) Train it to convergence



# Attempting to Train a Pruned Network

- 1) Train the network
- 2) Remove superfluous structure
- 3) Randomly initialize the pruned network
- 4) T

Does not reach full accuracy



# Attempting to Train a Pruned Network

Learning both Weights and Connections for Efficient Neural Networks	
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<b>John Tran</b> NVIDIA johntran@nvidia.com	<b>William J. Dally</b> Stanford University NVIDIA dally@stanford.edu

It is better to retain the weights from the initial training phase for the connections that survived pruning than it is to reinitialize.

# Attempting to Train a Pruned Network

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## PRUNING FILTERS FOR EFFICIENT CONVNETS

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It is better to retain the weights from the initial training phase for the connections that survived pruning than it is to reinitialize.

Training a pruned model from scratch performs worse than retraining a pruned model, which may indicate the difficulty of training a network with small capacity.

# Attempting to Train a Pruned Network

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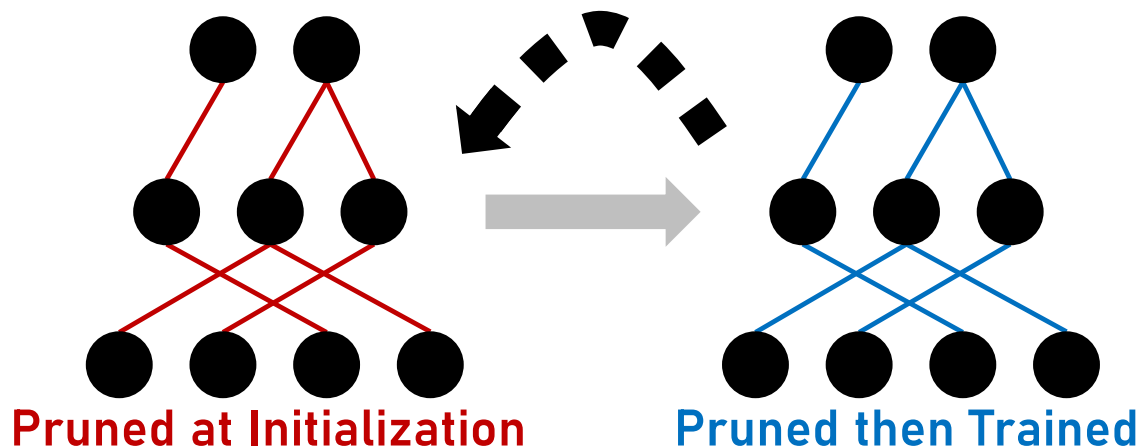
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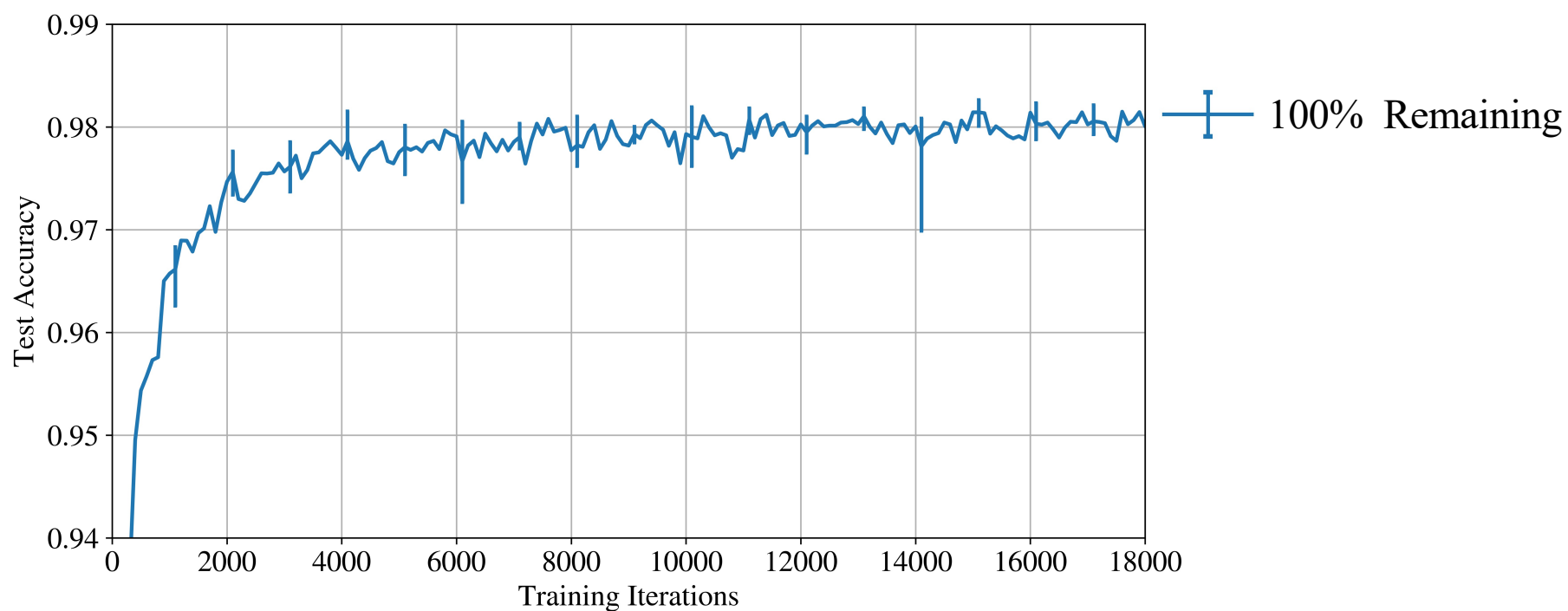


# Iterative Magnitude Pruning (IMP)

- 1) Train the network
- 2) Remove superfluous structure
- 3) Reset each weight to its original initialization
- 4) Train it to convergence and repeat iteratively



# Results

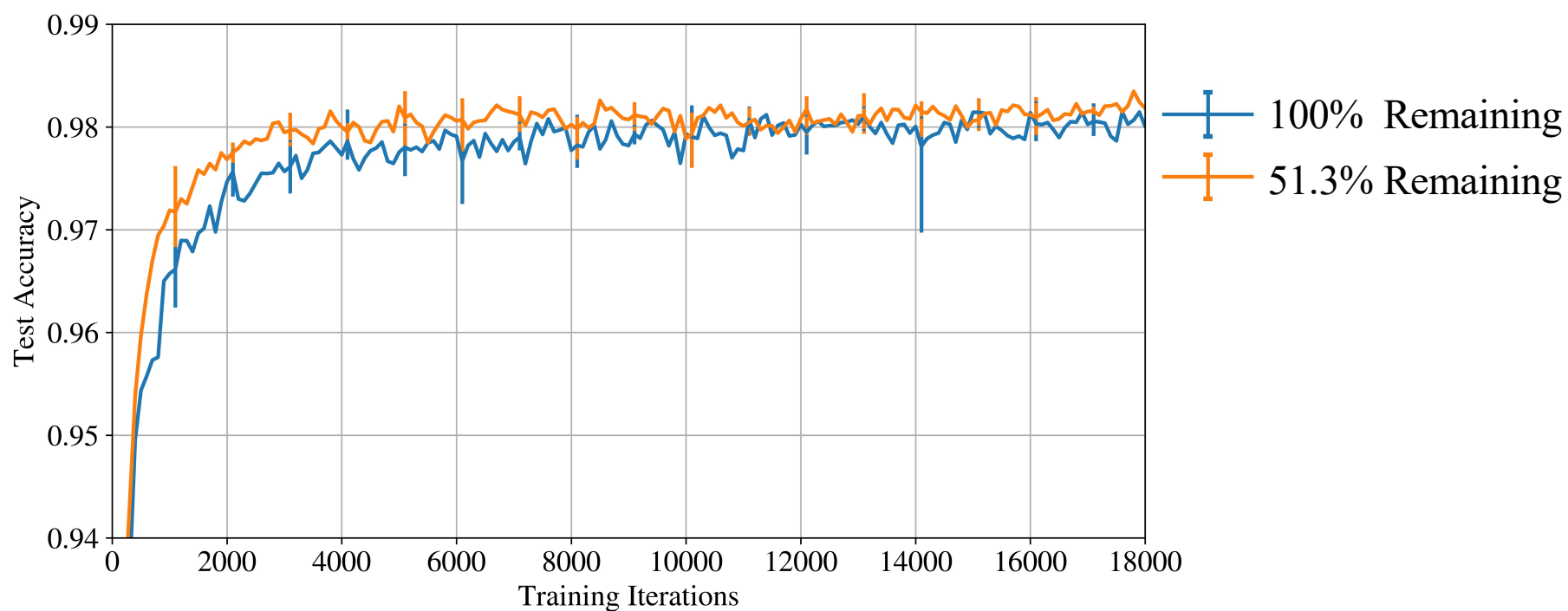


LeNet 300-100-10 for MNIST

fully-connected

300K parameters

# Results

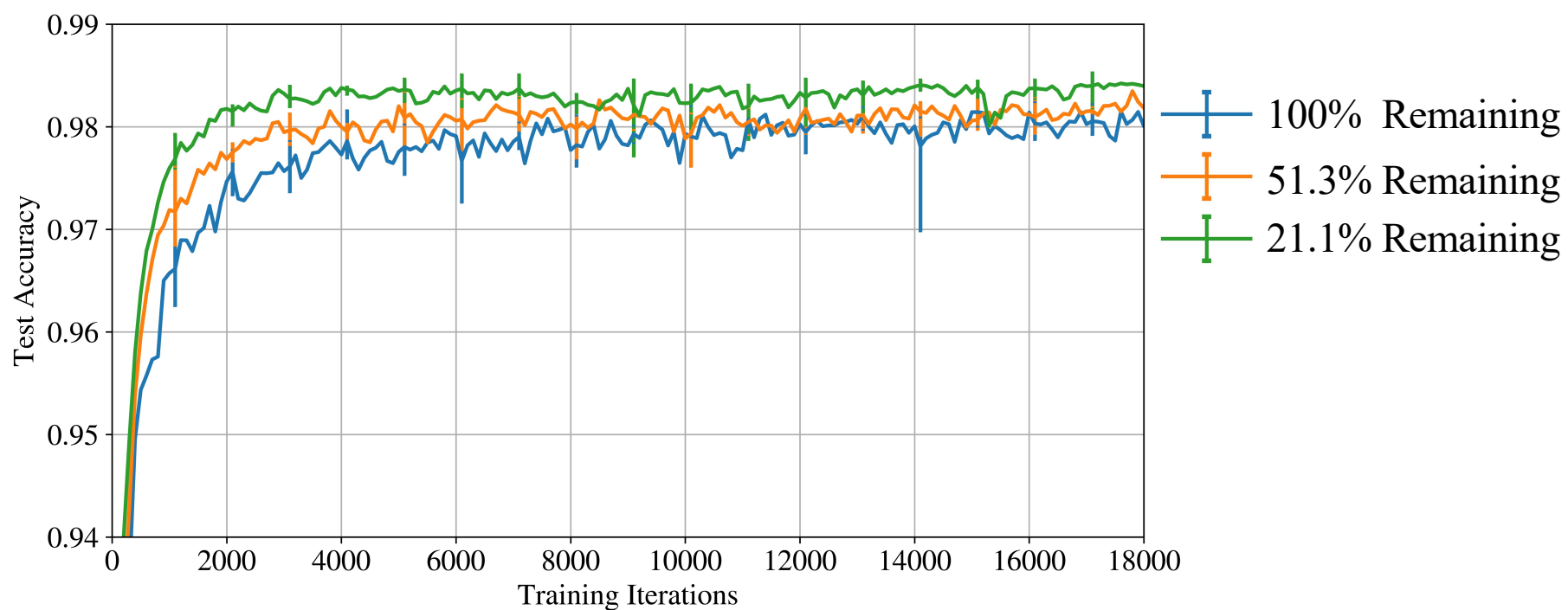


LeNet 300-100-10 for MNIST

fully-connected

300K parameters

# Results

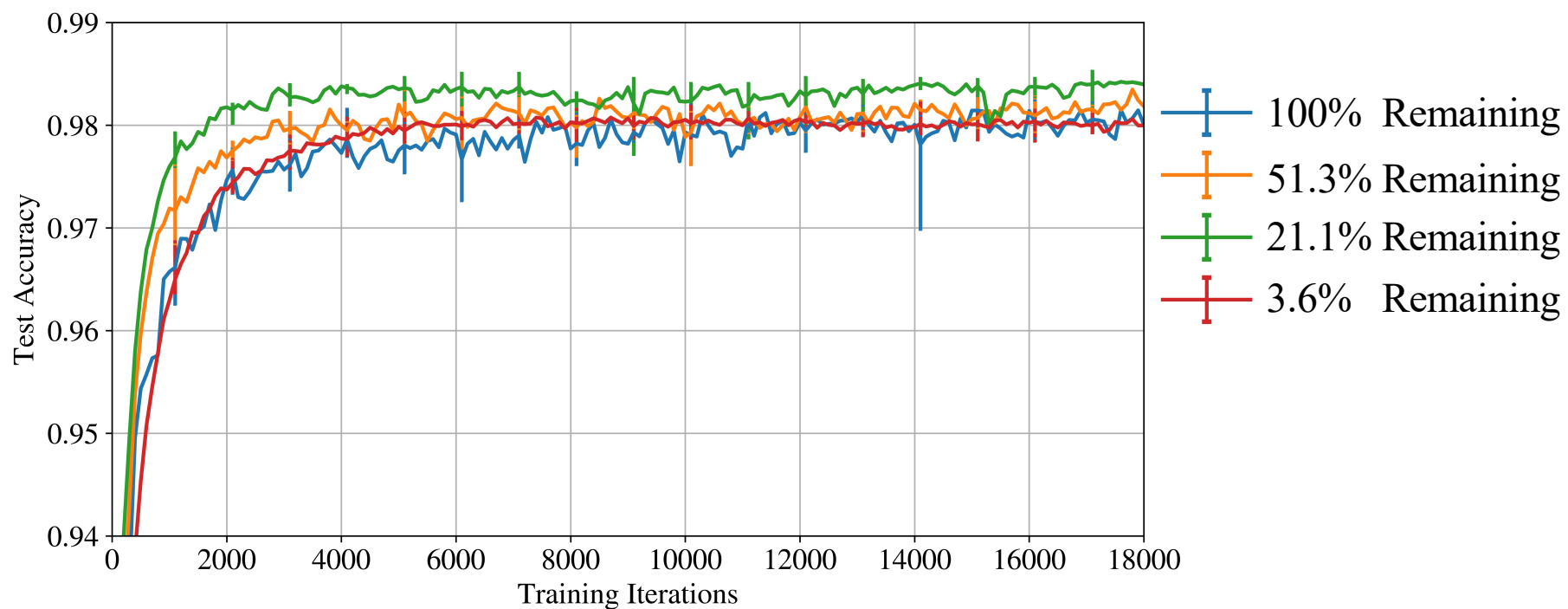


LeNet 300-100-10 for MNIST

fully-connected

300K parameters

# Results

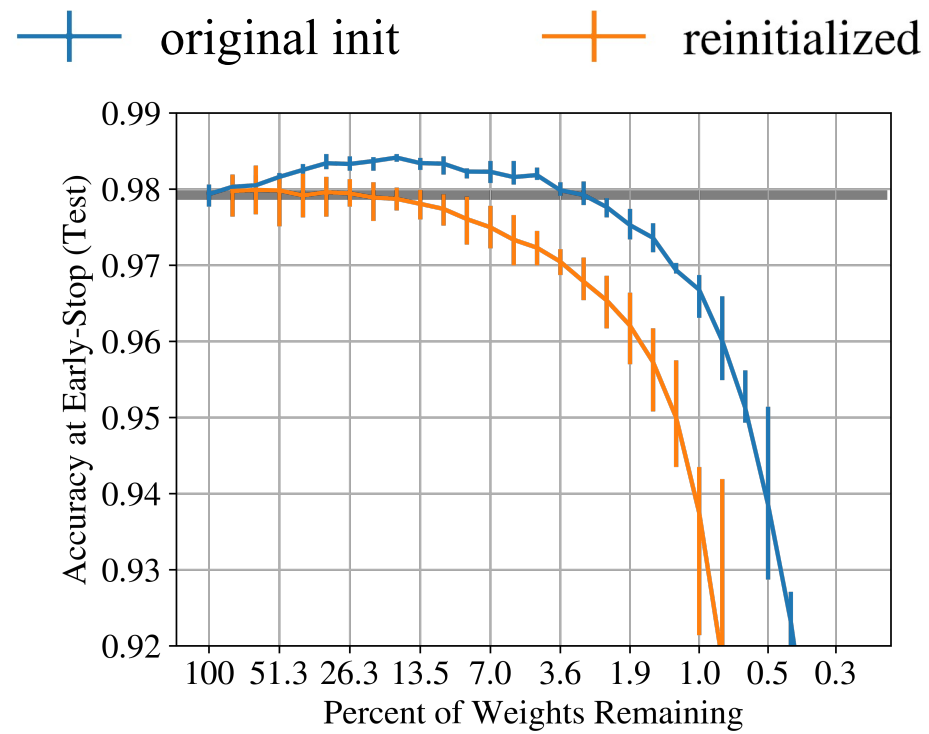


LeNet 300-100-10 for MNIST

fully-connected

300K parameters

# Results



LeNet 300-100-10 for MNIST

fully-connected

300K parameters

# Limitations

- Subnetworks are found retroactively
- Exploiting pruning for better performance requires software and hardware support
- Small, vision networks and tasks
  - Does not work for larger models and more complicated datasets

## Stay Tuned for the PC Meeting

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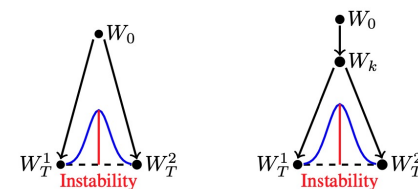
### Linear Mode Connectivity and the Lottery Ticket Hypothesis

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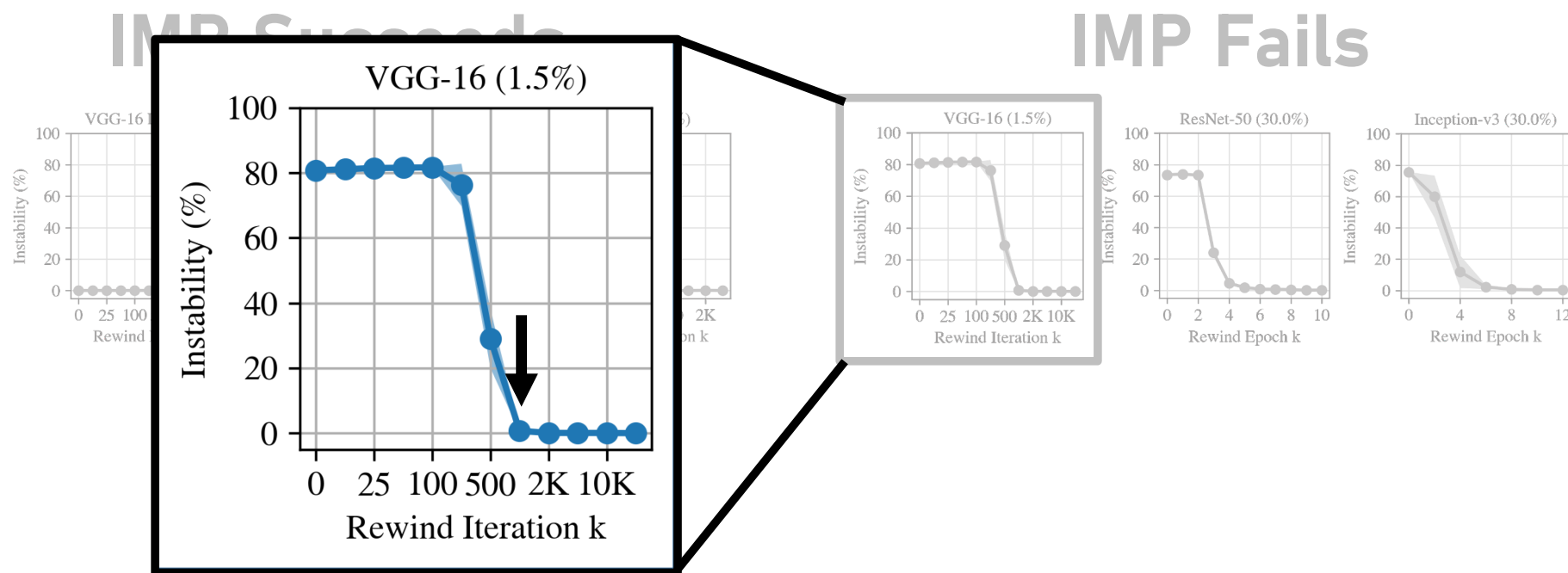
Jonathan Frankle<sup>1</sup> Gintare Karolina Dziugaite<sup>2</sup> Daniel M. Roy<sup>3,4</sup> Michael Carbin<sup>1</sup>

#### Abstract

Why does a neural network optimize to a linearly connected minimum under different samples of SGD noise (e.g., random data and augmentation)? We find that standard models become *stable to SGD noise* in this early in training. From then on, the outcome



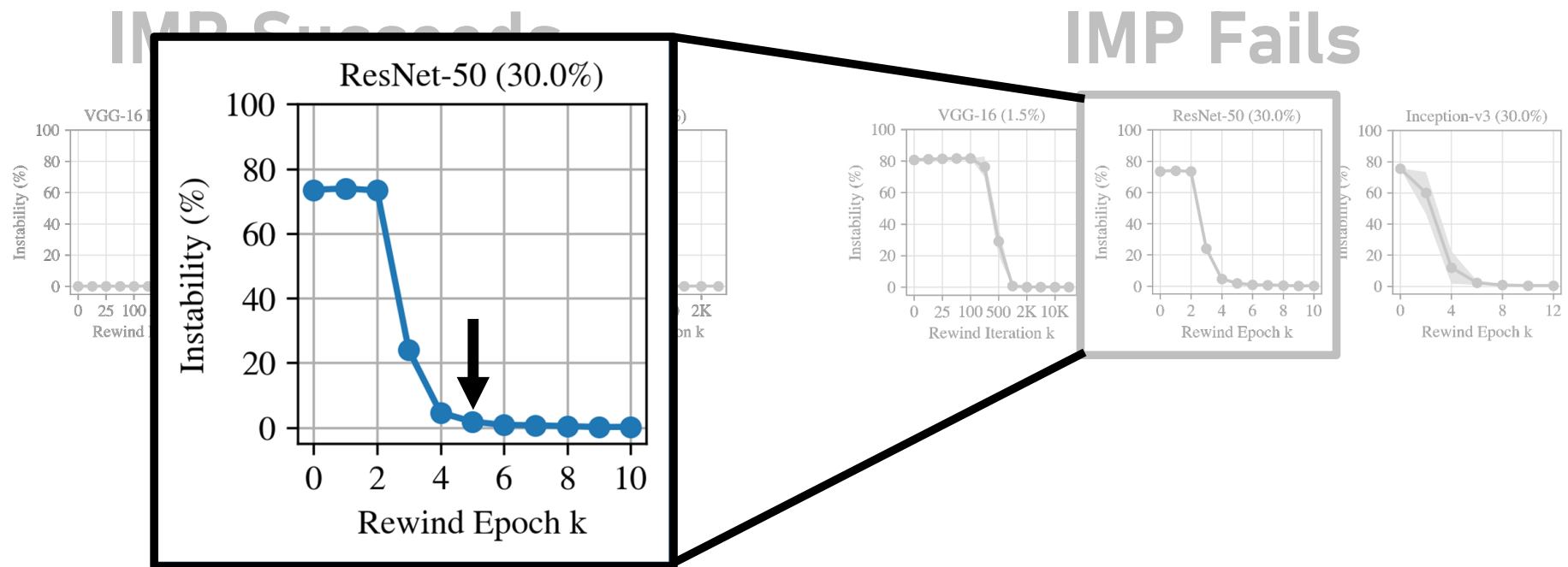
# Sneak Peak of Results For More Complex Models/Datasets



**Iteration 1K out of 62K = 1.6% into training**



# Sneak Peak of Results For More Complex Models/Datasets



**Epoch 5 out of 90 = 5.5% into training**

# High Level Findings in the literature on this topic

Sparse subnetworks reach full accuracy:

In small networks, this occurs at initialization.

For large networks, this occurs early in training.

(we will see the details during the PC meeting).

But this alone does not help us improve the speed up training. However, this is an active area of research, and if we could find the lottery hypothesis ticket in a more efficient manner, then this could significantly improve training cost.

# Another Limitation: Structured vs Unstructured Sparsity

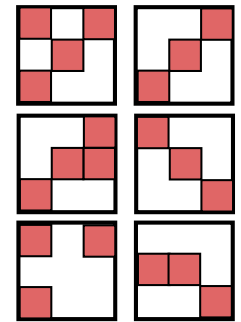
Another Limitation

## Unstructured Sparsity:

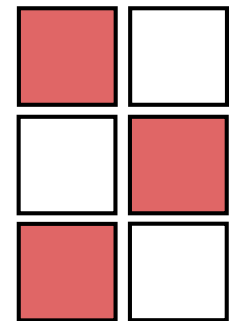
- Enables significantly higher levels of sparsity (~5%)
- Harder to accelerate in HW

## Structured Sparsity:

- Easier to accelerate in HW
- Does not allow as much sparsity (~60%)



Unstructured Sparsity



Structured Sparsity

# Further Reading

- We only talked about weight sparsity, but there is a large body of work on activation sparsity, token pruning, training free pruning, ...
- If you are interested to learn more about pruning:
- [Sparsity in Deep Learning: Pruning and growth for efficient inference and training in neural networks](#)
- [The state of sparsity in deep neural networks](#)
- [What is the State of Neural Network Pruning?](#)

# Efficient ML with Quantization

# Quantization Enables Fewer Memory Accesses

- Memory accesses are the principal cost in both latency and energy
- Lower precision weights in DNN mean each memory access brings more data values
- More data values few accesses overall

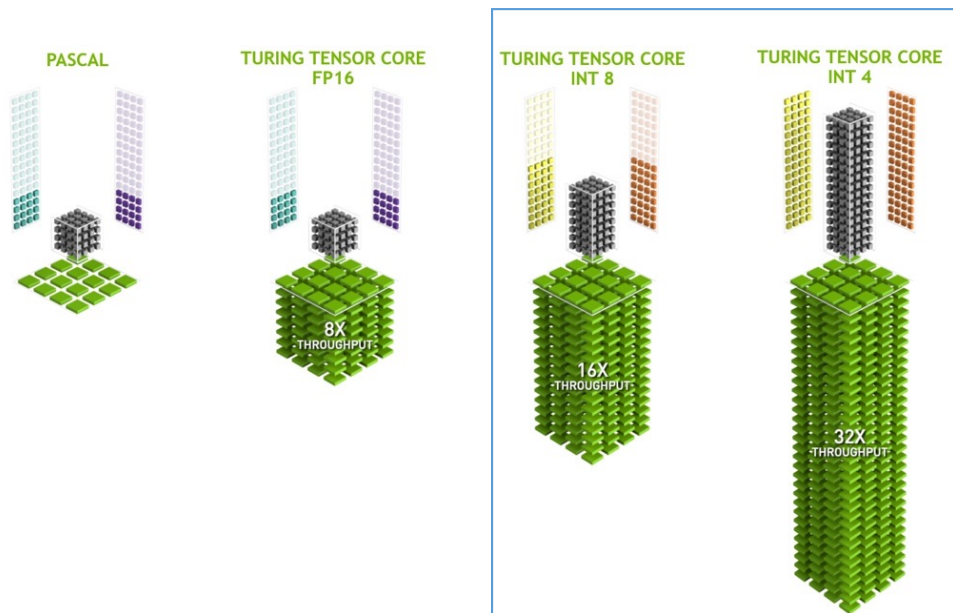


Image Credit: Sdxcentral, nvidia  
Table credit: Mark Horowitz

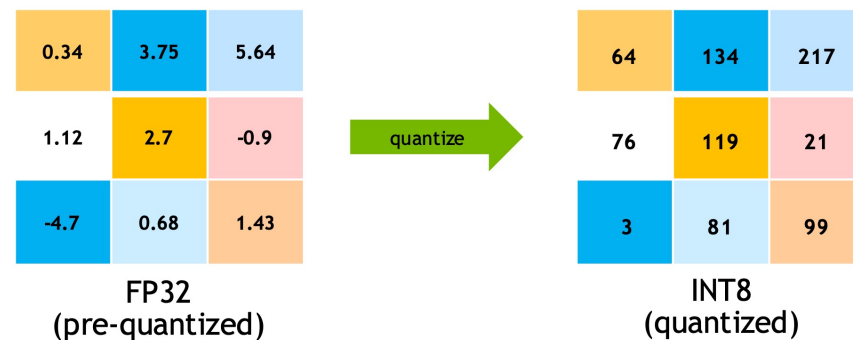
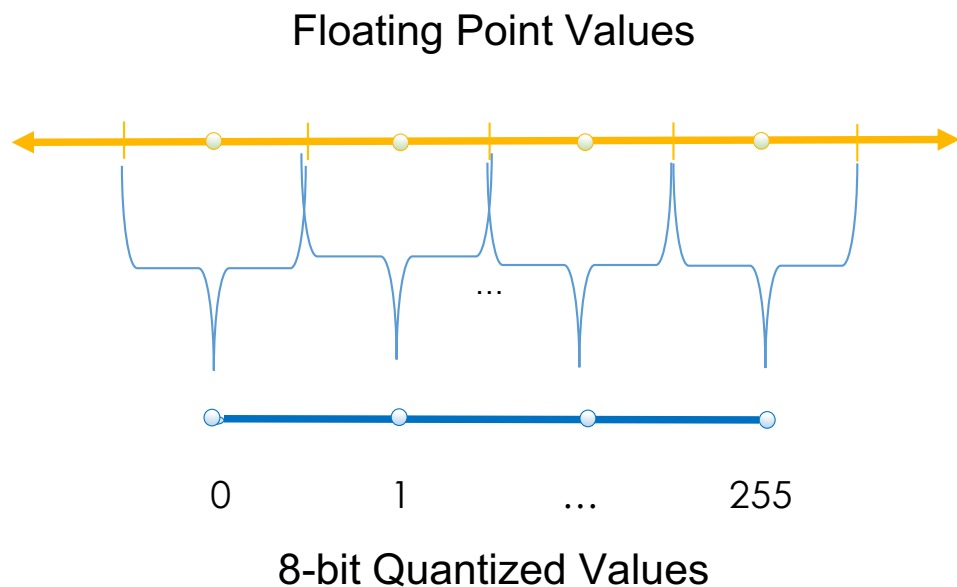
Operation:	Energy (pJ)	Relative Energy Cost
8b Add	0.03	
16b Add	0.05	
32b Add	0.1	
16b FP Add	0.4	
32b FP Add	0.9	
8b Multiply	0.2	
32b Multiply	3.1	
16b FP Multiply	1.1	
32b FP Multiply	3.7	
32b SRAM Read (8KB)	5	
32b DRAM Read	640	

[Horowitz, ISSCC 2014]

1 10 10<sup>2</sup> 10<sup>3</sup> 10<sup>4</sup>

# Quantization

- **Uniform quantization** is a linear mapping from floating point values to quantized integer values



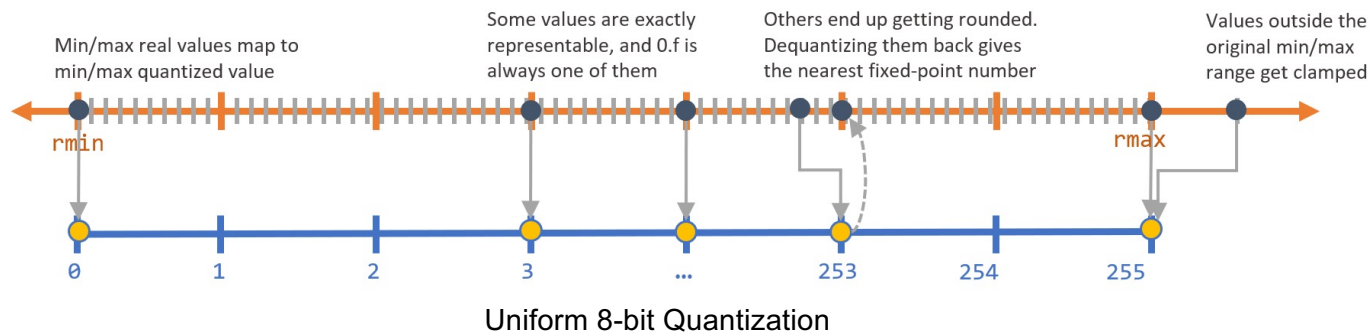
- **Less memory footprint**
- **Faster inference** on integer arithmetic units

# Uniform Quantization

- $r$ : Real value
- $r_{max}, r_{min}$ : max/min range of values
- $B$ : Quantization bits
- $S$  (FP32),  $z$  (int): Scale and bias
- $q$ : Fixed point quantized values

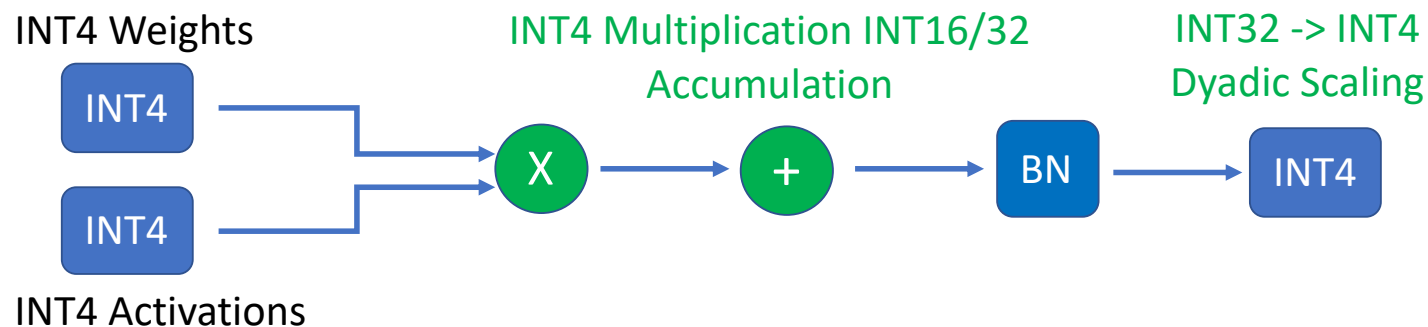
$$r = \frac{r_{max} - r_{min}}{2^B - 1}(q - z)$$

$$r = S(q - z)$$

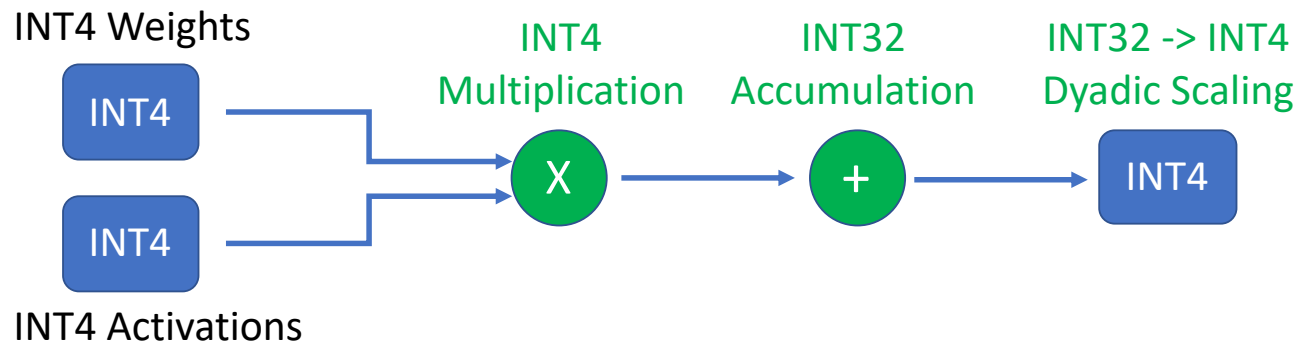




# Integer-only Quantization!



- Key idea is to not fold BN during training and only do BN folding at the end, otherwise you will effectively be training a model without BN which is known to be hard to optimize



# Integer-only Quantization Works

(a) *MobileNetV2*

Method	Int	Uni	BL	Precision	Size	BOPS	Top-1
Baseline	✗	✓	73.03	W32A32	13.2	322	73.03
RVQuant (Park et al., 2018)	✗	✗	70.10	W8A8	3.3	20	70.29
CalibTIB(Hubara et al., 2020)	✗	✓	71.90	W8A8	3.3	20	71.60
HAWQV3	✓	✓	73.03	W8A8	3.3	20	<b>73.01</b>

Almost not  
accuracy  
degradation

(c) *InceptionV3*

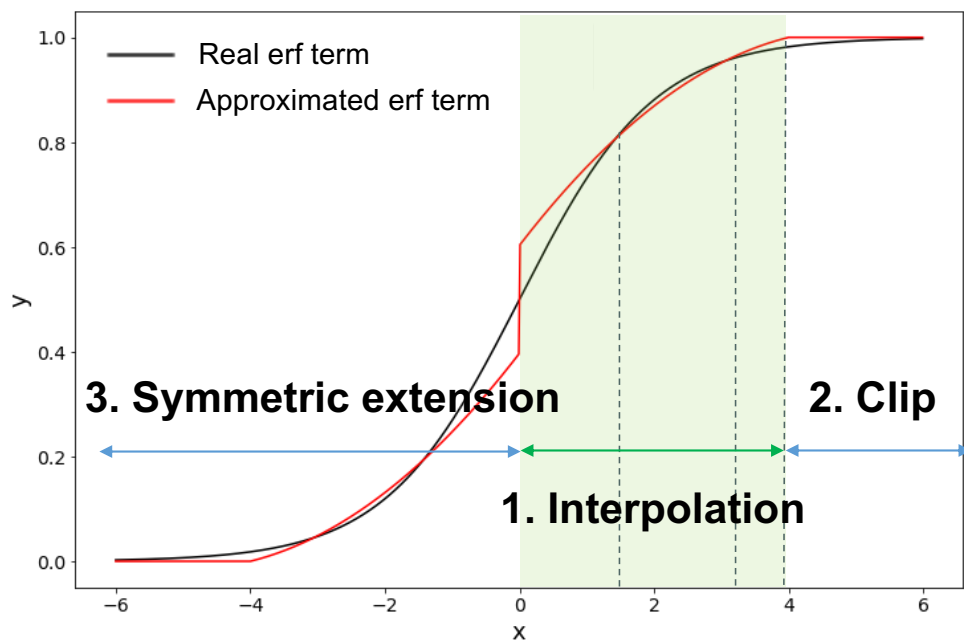
Method	Int	Uni	BL	Precision	Size	BOPS	Top-1
Baseline	✗	✓	78.88	W32A32	90.9	5850	78.88
Integer Only (Jacob et al., 2018)	✓	✓	78.30	W8A8	22.7	366	74.20
RVQuant (Park et al., 2018)	✗	✗	74.19	W8A8	22.7	366	74.22
HAWQV3	✓	✓	78.88	W8A8	22.7	366	<b>78.76</b>

Almost not  
accuracy  
degradation

Z. Yao\*, Z. Dong\*, Z. Zheng\*, A. Gholami\*, E. Tan, J. Li, L. Yuan, Q. Huang, Y. Wang, M. W. Mahoney, K. Keutzer, HAWQ-V3: Dyadic Neural Network Quantization in Mixed Precision, ICML'21.  
Online Code: <https://github.com/Zhen-Dong/HAWQ>

# Integer-only Can be applied to general Non-Linearities

- **Proposed Solution:** Approximate erf with 2<sup>nd</sup>-order polynomial



$$\text{GELU}(x) := x\Phi(x) = x \cdot \frac{1}{2} \left[ 1 + \overbrace{\text{erf}(x/\sqrt{2})}^{\text{Real erf term}} \right]$$

$$\text{i-GELU}(x) := x \cdot \frac{1}{2} \left[ 1 + \underbrace{L\left(\frac{x}{\sqrt{2}}\right)}^{\text{Approximated erf term}} \right]$$

Can be computed with integer arithmetic

# Integer-only Softmax

- We can even approximate  $\exp(x)$  with 2<sup>nd</sup>-order polynomials with very low error
- Approximate exponential in the interval  $[-\ln 2, 0]$

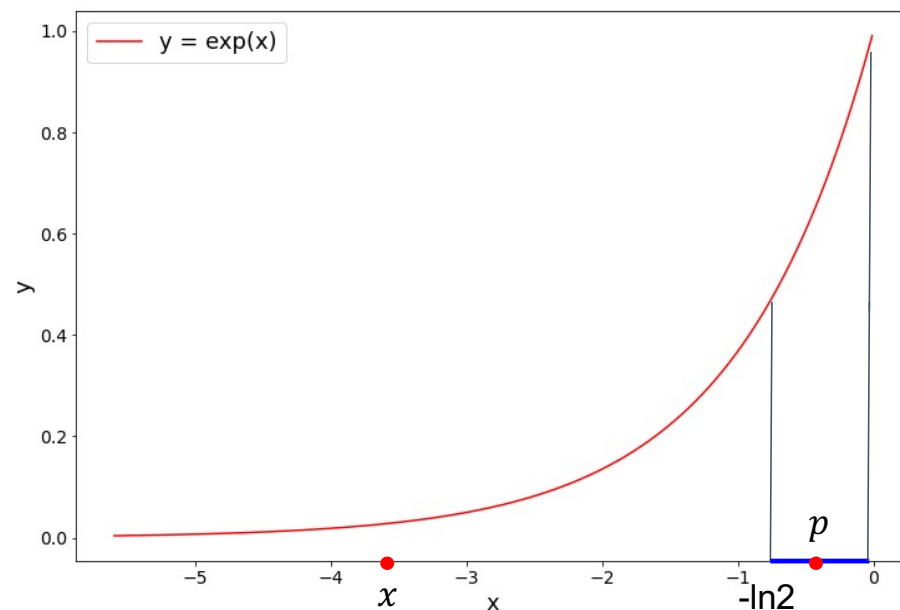
$$\tilde{x} = (-\ln 2)z + p$$

Quotient  $z$ : integer

Remainder  $p$ : lies in  $[-\ln 2, 0]$

$$\exp(\tilde{x}) = 2^{-z} \exp(p) = \exp(p) \gg z$$

$\underbrace{\exp(p)}_{\text{Compute with approximation}}$



# Integer-only Quantization Works: Transformers

(a) RoBERTa-Base

	Int-only	MNLI-m	MNLI-mm	QQP	QNLI	SST-2	CoLA	STS-B	MRPC	RTE	Avg.
Baseline	✗	87.8	87.4	90.4	92.8	94.6	61.2	91.1	90.9	78.0	86.0
I-BERT	✓	87.5	87.4	90.2	92.8	95.2	62.5	90.8	91.1	79.4	86.3
Diff		-0.3	0.0	-0.2	0.0	+0.6	+1.3	-0.3	+0.2	+1.4	+0.3

(b) RoBERTa-Large

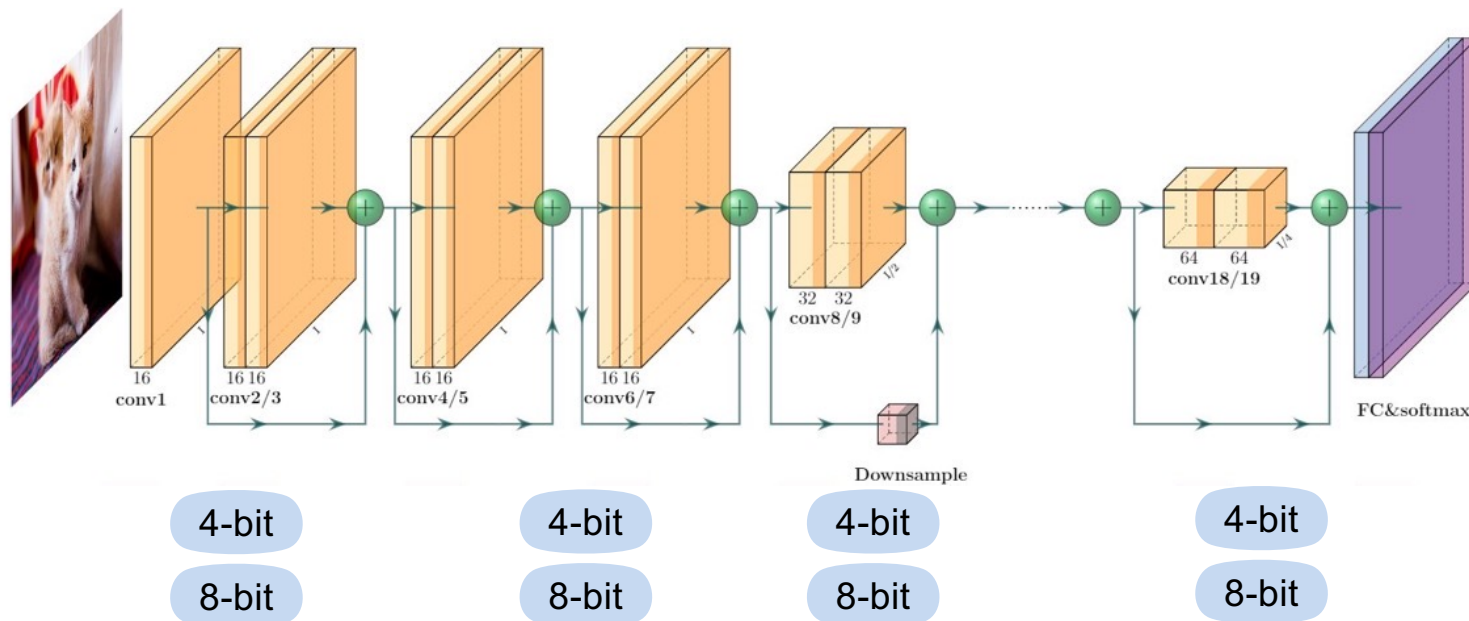
	Int-only	MNLI-m	MNLI-mm	QQP	QNLI	SST-2	CoLA	STS-B	MRPC	RTE	Avg.
Baseline	✗	90.0	89.9	92.8	94.1	96.3	68.0	92.2	91.8	86.3	89.0
I-BERT	✓	90.4	90.3	93.0	94.5	96.4	69.0	92.2	93.0	87.0	89.5
Diff		+0.4	+0.4	+0.2	+0.4	+0.1	+1.0	0.0	+1.2	+0.7	+0.5

➤ I-BERT is integrated in **HuggingFace** ([~8K downloads per month](#))

S. Kim\*, A. Gholami\*, Z. Yao\*, M. Mahoney, K. Keutzer, **I-BERT: Integer-only BERT Quantization**, ICML, 2021.

# What about Sub-INT8 Quantization?

Can we go even further and perform lower precision quantization?



Uniform low precision does not work as it can significantly degrade accuracy => **Use mixed-precision**

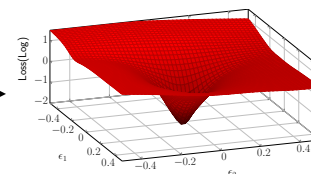
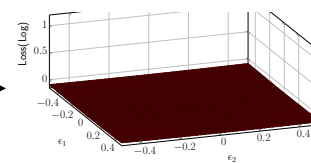
- **But how should we set the precision for each kernel?**

# Hessian Aware Quantization

This is somewhat similar to the **Jenga** game. We only remove blocks that are not sensitive.

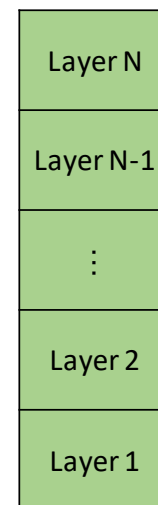
- Only use low precision quantization for insensitive parameters (flat loss landscape)
- Use high precision quantization for sensitive parameters (sharp loss landscape)

This sensitivity can be calculated through Hessian which quantifies the relative sharpness/flatness of the loss landscape.

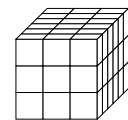
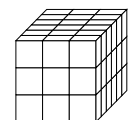
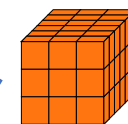


[Image from UniversityCoop](#)

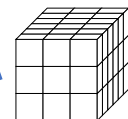
Output:  $\hat{y}$



Input:  $x$



⋮



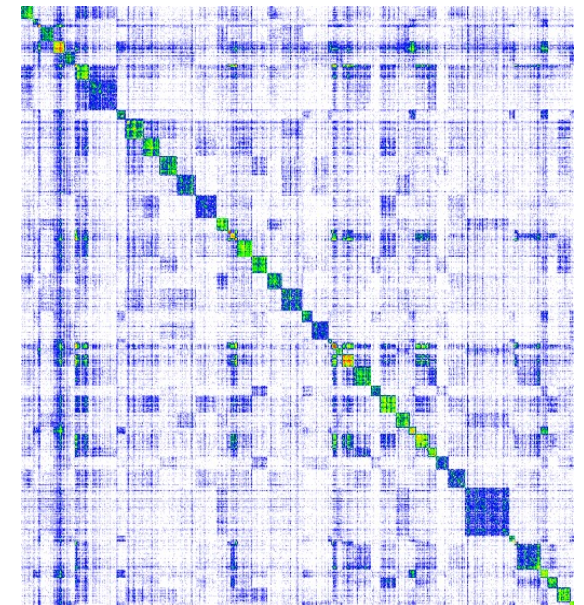
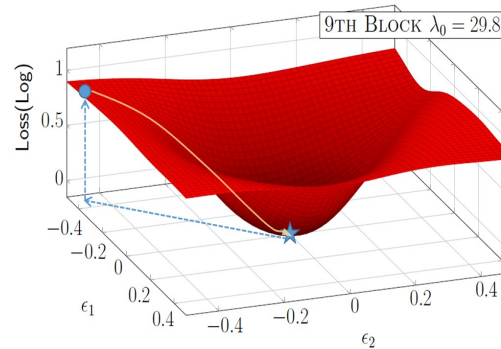
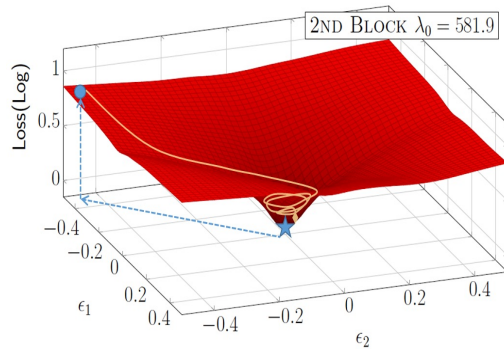
Dong Z, Yao Z, Arfeen D, Gholami A, Mahoney MW, Keutzer K. Hawq-v2: Hessian aware trace-weighted quantization of neural networks. NeurIPS, 2020.  
Yu S, Yao Z, Gholami A, Dong Z, Mahoney MW, Keutzer K. Hessian-Aware Pruning and Optimal Neural Implant. WACV, 2022.



# Hessian and Loss Landscape Curvature

**Theorem 1** *After quantizing the model to same precision, fine tuning layers that have smaller average trace of Hessian can achieve a smaller loss, compared to layers with larger average trace of Hessian.*

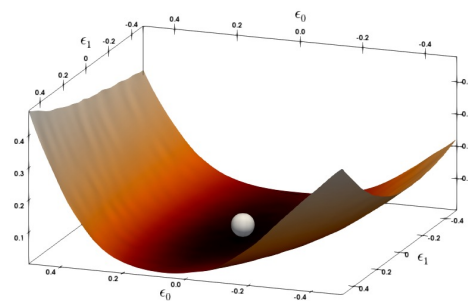
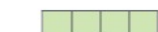
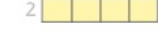
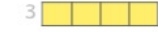
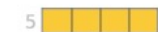
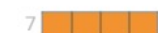
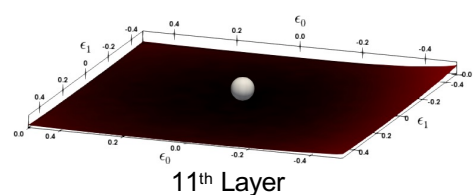
$$\text{average } \text{tr}(H) = \frac{1}{n} \sum_i H_{ii} = \frac{1}{n} \sum_i \lambda_i(H)$$



Z. Dong, Z. Yao, D. Arfeen, A. Gholami, M. Mahoney, K. Keutzer, HAWQ-V2: Trace Weighted Hessian Aware Quantization, **NeurIPS 2020**.  
Z. Yao\*, Z. Dong\*, Z. Zheng\*, A. Gholami\*, E. Tan, J. Li, L. Yuan, Q. Huang, Y. Wang, M. W. Mahoney, K. Keutzer, HAWQ-V3: Dyadic Neural Network Quantization in Mixed Precision, **ICML, 2021**.



# Mixed Precision Quantization for NLP

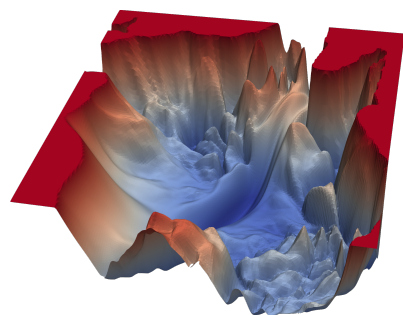


## Named Entity Recognition Task

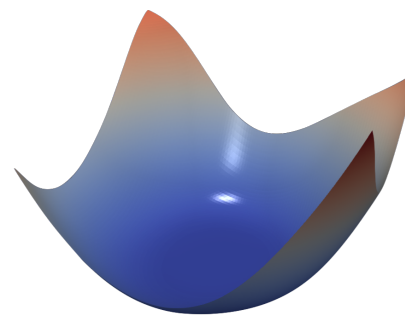
Method	w-bits	e-bits	F <sub>1</sub>	Size
Baseline	32	32	95.00	410.9
Q-BERT	8	8	94.79	102.8
DirectQ	4	8	89.86	62.2
Q-BERT	4	8	<b>94.90</b>	62.2
DirectQ	3	8	84.92	52.1
Q-BERT	3	8	<b>94.78</b>	52.1
Q-BERT <sub>MP</sub>	2/4 <sub>MP</sub>	8	<b>94.55</b>	52.1
DirectQ	2	8	54.50	42.0
Q-BERT	2	8	<b>91.06</b>	42.0
Q-BERT <sub>MP</sub>	2/3 <sub>MP</sub>	8	<b>94.37</b>	45.0

# Summary:

- Quantization and Pruning of a model depends on the loss landscape
- INT8 Quantization has been proved to be very effective for inference without accuracy degradation
  - Typically yields 2-4x speed up and 4x reduction in memory volume
- Lower Precision Quantization is possible but often there is an accuracy/speed trade-off



Less Robust to Compression



More Robust to Compression

# Further Reading

- If you are interested to learn more about quantization see the following papers:
- [A survey of quantization methods for efficient neural network inference](#)
- [Quantization and training of neural networks for efficient integer-arithmetic-only inference](#)